Modulational instability of beat waves in a transversely magnetized plasma: Ion effects

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The effect of ion dynamics on the modulational instability of the electrostatic beat wave at the difference frequency of two incident laser beams in a hot, collisionless, and transversely magnetized plasma has been studied theoretically. The full Vlasov equation in terms of gyrokinetic variables is employed to obtain the nonlinear response of ions and electrons. It is found that the growth rate of modulational instability is about two orders higher when ion motions are included.

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I. INTRODUCTION

Recently there has been a great interest in the generation of large amplitude electron plasma waves at the beat frequency of two high power laser radiation in a plasma [1–4]. The most promising application of beat waves is in the plasma beat wave accelerator (PBWA) [5–9]. Other applications of beat waves include plasma heating and current drive [10,11], plasma diagnostics [12], etc. Recently, in the case of particle acceleration, Modena *et al.* [13] experimentally observed the acceleration of electrons at the wave-breaking limit of a nonlinear relativistic electron plasma wave excited by strong forward Raman scattering induced by a high-intensity ($>5\times10^{18}$ W cm^{-2}) short-pulse laser. However, the excitation of the electron plasma wave and the acceleration of electrons can be controlled more efficiently in the well known plasma beat wave accelerator scheme. Amiranoff *et al.* [14] observed appreciable acceleration of electrons in a recent experiment on the PBWA. Before achieving the possible acceleration of particles to ultrahigh energy, the longitudinal large amplitude electron plasma wave may couple parametrically with different plasma modes, and may suffer a number of strong microinstabilities which may deteriorate the acceleration process seriously. Amiranoff *et al.* [14] concluded from their experiment that, in particular, modulational instability can seriously destroy the excited plasma wave and stop the acceleration mechanism. Therefore, the study of the parametric instabilities of beat waves and their possible saturation is of great importance in the PBWA.

It is well known that the coupling of the Langmuir wave to the ion motion can give rise either to modulational or decay instabilities [15]. On a slow time scale, ion motion [16] can play an important role in saturating the large amplitude beat wave. In the present study we retain the ion dynamics as well as electron dynamics in the nonlinear response, because the time scale of the ion motion ω_{pi}^{-1} (where ω_{pi} is the ion plasma frequency), which is very short on the order of a few picoseconds at the plasma density 10^{16}–10^{17} cm^{-3}, can be less or comparable to the duration of the laser pulses in the present day experiments [14,17]. Therefore the role of ions may be important in these experiments. However, at higher pump strength and for short pulses (less than an ion period), instabilities, particularly those which involve ion motion, can be avoided [9]. To the best knowledge of the authors, there has been no study of beat wave instabilities including the ion dynamics.

In the surfatron scheme [18], a potential modification of the PBWA, an external transverse static magnetic field is applied to phase lock the particles with the wave for preventing the accelerated particles from outrunning the plasma wave, thereby eliminating the limitation on the maximum energy gain of the plasma particles. In the presence of an external magnetic field, a plasma supports a variety of plasma modes. The plasma wave in the PWBA attains a large amplitude, and may couple parametrically to these modes, deteriorating the uniform acceleration of particles to high energy. Therefore, a detailed investigation of all possible parametric instabilities of this high amplitude electron plasma wave in the presence of an external transverse static magnetic field is of great importance in the context of the PBWA. In this paper, we have studied, in particular, the four wave parametric instability, i.e., the modulational instability of a large amplitude longitudinal beat wave excited at the beat frequency of two high power electromagnetic waves in a homogeneous, hot, and transversely magnetized plasma including ion dynamics. We consider short-wavelength perturbations which may be present in the laser-produced plasma because of the temperature or density gradients caused by the presence of ion/electron plasma waves.

The plan of the paper is as follows. In Sec. II, we study the response of magnetized ions and electrons of the plasma by employing the full Vlasov equation in terms of gyrokinetic variables for the low-frequency mode. In the case of the plasma beat wave accelerators, the condition $\omega_{pe}^2 \gg \omega_{ce}^2$ (where ω_{pe} and ω_{ce} are, respectively, the electron plasma frequency and electron cyclotron frequency) is always satisfied. We may therefore consider the high-frequency response of electrons to be unmagnetized. The frequency of the low-frequency mode is usually less than frequency of the electron plasma wave excited at the beat frequency of the two incident laser beams, i.e., the pump waves. The Larmor radii of electrons and ions corresponding to this low-frequency mode may be larger than or comparable to the wavelengths of these waves. Hence we employ the kinetic equation for the nonlinear response of electrons and ions in the plasma. In Sec. III, the growth rate of the four wave decay process, i.e., the modulational instability, has been obtained. In this case the phase velocity of the low-frequency-driven mode is considered to be equal to the group velocity of the beat wave. A

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numerical estimate of our results is given in Sec. IV. Finally, a brief discussion is presented in Sec. V.

II. NONLINEAR RESPONSE OF ELECTRONS AND IONS

We consider the propagation of two colinear high-amplitude upper-hybrid electromagnetic waves $(\omega_1', \mathbf{k}_1')$ and $(\omega_2', \mathbf{k}_2')$ in a transversely magnetized $(\mathbf{B}_s \| \hat{z})$, hot, homogeneous, and collisionless plasma:

$$\mathbf{E}_{1,2}' = \mathbf{E}_{1,2}'' \exp[-i(\omega_{1,2}' t - k_{1,2}' x)], \tag{1}$$

where

$$k_{1,2}' = \frac{\omega_{1,2}'}{c}\left[1 - \frac{\omega_{\rm pe}^2}{\omega_{1,2}'^2}\frac{\omega_{1,2}'^2 - \omega_{\rm pe}^2}{\omega_{1,2}'^2 - \omega_{\rm pe}^2 - \omega_{\rm ce}^2}\right]^{1/2},$$

$$\omega_{\rm pe} = \left(\frac{4\pi e^2 n_0^0}{m_e}\right)^{1/2}, \tag{2}$$

$$\omega_{\rm ce} = \frac{eB_s}{m_e c}.$$

Here $-e$, m_e, n_0^0, and c are the electronic charge, mass, unperturbed equilibrium electron density, and velocity of light in a vacuum, respectively. On account of the nonlinear interaction of the incident electromagnetic waves in the plasma, a large-amplitude longitudinal electrostatic electron plasma wave $(\omega_0, \mathbf{k}_0; \omega_0 = \omega_1' - \omega_2',\ \mathbf{k}_0 = \mathbf{k}_1' - \mathbf{k}_2')$ is generated at the difference frequency:

$$\begin{aligned}\mathbf{E}_0(\omega_0, \mathbf{k}_0) &= -i\mathbf{k}_0 \phi_0(\omega_0, \mathbf{k}_0) \\ &= -\hat{\mathbf{x}} i k_0 \phi_0 \exp[-i(\omega_0 t - k_0 x)],\end{aligned} \tag{3}$$

where ϕ_0 is the electrostatic potential of the beat wave (ω_0, k_0) which satisfies the Bohm-Gross dispersion relation for the magnetized plasma,

$$\omega_0^2 = \omega_{\rm pe}^2 + \omega_{\rm ce}^2 + 3k_0^2 v_{{\rm th},e}^2/2, \tag{4}$$

$v_{{\rm th},e} = (2T_e/m_e)^{1/2}$ is the thermal speed of electrons, and T_e is the temperature of the plasma electrons measured in units of the Boltzmann constant.

Now we consider that this longitudinal electron plasma wave will interact with a short-wavelength low-frequency electrostatic density perturbation associated with a plasma mode $(\omega, \mathbf{k})$, and generate two high-frequency sideband modes $(\omega_{1,2} = \omega \mp \omega_0,\ \mathbf{k}_{1,2} = \mathbf{k} \mp \mathbf{k}_0)$. The waves $(\omega, \mathbf{k})$, $(\omega_1, \mathbf{k}_1)$, and $(\omega_2, \mathbf{k}_2)$ will grow at the expense of the energy from the pump wave $(\omega_0, \mathbf{k}_0)$. For the hot magnetized plasma, the Larmor radii of electrons and ions may be larger than any of the wavelengths of the waves involved, i.e., $k_0\rho_e$, $k\rho_e$, $k_1\rho_e$, $k_2\rho_e$, $k_0\rho_i$, $k\rho_i$, $k_1\rho_i$, and $k_2\rho_i \geqslant 1$, where $\rho_e = v_\perp/\omega_{\rm ce}$ and $\rho_i = v_\perp/\omega_{\rm ci}$. The symbol $\perp$ denotes quantities perpendicular to the external magnetic field. Hence the fluid model of plasma breaks down, and one must solve the full Vlasov equation for the nonlinear response of electrons and ions in the plasma [19].

A. Response of ions

We study the response of ions to the four-wave decay process in the presence of an external static magnetic field by the nonlinear Vlasov equation expressed in terms of the guiding center coordinates $\mathbf{x}_g$, magnetic moment μ, and the polar angle θ of perpendicular velocity and parallel momentum $p_\|$:

$$\frac{\partial f_i^T}{\partial t} + \dot{\mathbf{x}}_g \cdot \frac{\partial f_i^T}{\partial \mathbf{x}_g} + \dot{\mu}\frac{\partial f_i^T}{\partial \mu} + \dot{\theta}\frac{\partial f_i^T}{\partial \theta} + \dot{p}_\| \frac{\partial f_i^T}{\partial p_\|} = 0, \tag{5}$$

where

$$x_g = x + \rho_i \sin\theta,$$

$$y_g = y - \rho_i \cos\theta, \tag{6}$$

$$z_g = z.$$

$\omega_{\rm ci} = eB_s/m_i c$, $\mu = m_i v_\perp^2/2\omega_{\rm ci}$, the superscript T refers to the total quantity, and the symbol $\|$ denotes quantities parallel to the external magnetic field; the dot over a quantity denotes derivative with respect to time. Using equations of motion, we can easily deduce

$$\dot{\mu} = \frac{e}{\omega_{\rm ci}}\mathbf{E}_\perp^T \cdot \boldsymbol{v}_\perp = \frac{\partial H}{\partial \theta}, \tag{7}$$

$$\dot{\theta} = -\frac{\partial H}{\partial \mu} = -\left[\omega_{\rm ci} + \frac{e}{m_i v_\perp}(E_x^T \sin\theta - E_y^T \cos\theta)\right], \tag{8}$$

$$\dot{\mathbf{x}}_g = \boldsymbol{v}_\| + \frac{e}{m_i \omega_{\rm ci}^2}\mathbf{E}_\perp^T \times \boldsymbol{\omega}_{\rm ci}, \tag{9}$$

where

$$H = \mu\omega_{\rm ci} + \frac{p_\|^2}{2m_i} + e\phi^T \tag{10}$$

is the Hamiltonian, and

$$\phi^T = \phi_0(\omega_0, \mathbf{k}_0) + \phi(\omega, \mathbf{k}) + \phi_1(\omega_1, \mathbf{k}_1) + \phi_2(\omega_2, \mathbf{k}_2) \tag{11}$$

is the total electrostatic potential in the system. Since (μ, θ), (x_g, y_g), and $(p_\|, z)$ form the canonical set of variables, Eq. (5) follows directly from the continuity equation of ion density in the six dimensional space of the resulting variables [19].

In the presence of the electrostatic potentials of the pump and the decay waves, the total distribution function of ions in Eq. (5) may be decomposed as

$$f_i^T = f_{0i}^0 + f_{0i}(\omega_0, \mathbf{k}_0) + f_i(\omega, \mathbf{k}) + f_{1i}(\omega_1, \mathbf{k}_1) + f_{2i}(\omega_2, \mathbf{k}_2), \tag{12}$$

where the space and time variations are implied, and the equilibrium distribution function f_{0i}^0 is taken to be Maxwellian at the temperature T_i:

$$f_{0i}^0 = n_0^0 \left(\frac{m_i}{2\pi T_i} \right)^{3/2} \exp\left(-\frac{m_i v^2}{2T_i} \right). \tag{13}$$

f_{0i}, f_{1i}, and f_{2i} are the high-frequency response (at the pump and sideband frequencies), and f_i is the low-frequency response. Using the identity

$$\exp[-i(\omega t - \mathbf{k}\cdot\mathbf{x})] \equiv \exp[-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)] \times \sum_n \exp[-in(\theta-\delta)] J_n(k_\perp \rho), \tag{14}$$

where J_n is the Bessel function of order n and the summation over n runs from $-\infty$ to $+\infty$, we can express

$$\mathbf{E}^T = -i\mathbf{k}_0\phi_0 \exp[-i(\omega_0 t - k_0 x_g)] \sum_n \exp(-in\theta) J_n^0 - i\mathbf{k}\phi \exp[-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta)] J_n - i\mathbf{k}_1\phi_1 \exp[-i(\omega_1 t - \mathbf{k}_1\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta_1)] J_n^1 - i\mathbf{k}_2\phi_2 \exp[-i(\omega_2 t - \mathbf{k}_2\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta_2)] J_n^2, \tag{15}$$

$$f_i^T = f_{0i}^0 + \exp[-i(\omega_0 t - k_0 x_g)] \sum_n \exp(-in\theta) f_{\mathrm{ni}}^0 + \exp[-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta)] f_{\mathrm{ni}} + \exp[-i(\omega_1 t - \mathbf{k}_1\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta_1)] f_{\mathrm{ni}}^1 + \exp[-i(\omega_2 t - \mathbf{k}_2\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta_2)] f_{\mathrm{ni}}^2. \tag{16}$$

In Eq. (15), ϕ's are the amplitudes of the electrostatic potentials of the waves, $J_n = J_n(k_\perp \rho_i)$, $J_n^0 = J_n(k_0\rho_i)$, $J_n^1 = J_n(k_{1\perp}\rho_i)$, and $J_n^2 = J_n(k_{2\perp}\rho_i)$, where δ, δ_1, and δ_2 are the angles between the x axis and $\mathbf{k}_\perp$, $\mathbf{k}_{1\perp}$, and $\mathbf{k}_{2\perp}$, respectively. Using Eqs. (15) and (16) into Eqs. (7)–(9), we can write

$$\dot{\mu} = -ie\phi_0 \exp[-i(\omega_0 t - k_0 x_g)] \sum_n n \exp(-in\theta) J_n^0 - ie\phi \exp[-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)] \sum_n n \exp[-in(\theta-\delta)] J_n - ie\phi_1 \exp[-i(\omega_1 t - \mathbf{k}_1\cdot\mathbf{x}_g)] \sum_n n \exp[-in(\theta-\delta_1)] J_n^1 - ie\phi_2 \exp[-i(\omega_2 t - \mathbf{k}_2\cdot\mathbf{x}_g)] \sum_n n \exp[-in(\theta-\delta_2)] J_n^2, \tag{17}$$

$$\dot{\theta} = -\omega_{ci} - \frac{e\phi_0 k_0}{m_i v_\perp} \exp[-i(\omega_0 t - k_0 x_g)] \sum_n \exp(-in\theta) J_n^{0\prime} - \frac{e\phi k_\perp}{m_i v_\perp} \exp[-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta)] J_n' - \frac{e\phi_1 k_{1\perp}}{m_i v_\perp} \exp[-i(\omega_1 t - \mathbf{k}_1\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta_1)] J_n^{1\prime} - \frac{e\phi_2 k_{2\perp}}{m_i v_\perp} \exp[-i(\omega_2 t - \mathbf{k}_2\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta_2)] J_n^{2\prime}, \tag{18}$$

$$\dot{x}_g = -\frac{ie}{m_i\omega_{\mathrm{ci}}} \Big[k_\perp \phi \sin\delta \exp[-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta)] J_n + k_{1\perp}\phi_1 \sin\delta_1 \exp[-i(\omega_1 t - \mathbf{k}_1\cdot\mathbf{x}_g)] \times \sum_n \exp[-in(\theta-\delta_1)] J_n^1 + k_{2\perp}\phi_2 \sin\delta_2 \exp[-i(\omega_2 t - \mathbf{k}_2\cdot\mathbf{x}_g)] \sum_n \exp[-in(\theta-\delta_2)] J_n^2 \Big], \tag{19}$$

$$\dot{y}_g=\frac{ie}{m_i\omega_{\rm ci}}\bigg[k_0\phi_0\exp[-i(\omega_0t-k_0x_g)]\sum_n\exp(-in\theta)J_n^0+k_\perp\phi\cos\delta\exp[-i(\omega t-\mathbf{k}\cdot\mathbf{x}_g)]\sum_n\exp[-in(\theta-\delta)]J_n$$
$$+k_{1\perp}\phi_1\cos\delta_1\exp[-i(\omega_1t-\mathbf{k}_1\cdot\mathbf{x}_g)]\sum_n\exp[-in(\theta-\delta_1)]J_n^1$$
$$+k_{2\perp}\phi_2\cos\delta_2\exp[-i(\omega_2t-\mathbf{k}_2\cdot\mathbf{x}_g)]\sum_n\exp[-in(\theta-\delta_2)]J_n^2\bigg], \quad (20)$$

$$\dot{z}_g=\frac{p_\parallel}{m_i}, \quad (21)$$

where the prime on the Bessel functions denotes a derivative with respect to its argument. Now, since the maximum growing modes propagate in the plane perpendicular to the external magnetic field [19,20] we take $k_\perp \gg k_\parallel$. Using Eqs. (15)–(21) in the Vlasov equation (5), we obtain the following linear response of ions:

$$f_{\rm ni}^0=\frac{e\phi_0}{T_i}\,\frac{n\omega_{\rm ci}}{\omega_0-n\omega_{\rm ci}}J_n^0f_{0i}^0,$$
$$f_{\rm ni}=\frac{e\phi}{T_i}\,\frac{n\omega_{\rm ci}}{\omega-n\omega_{\rm ci}}J_nf_{0i}^0,$$
$$f_{\rm ni}^1=\frac{e\phi_1}{T_i}\,\frac{n\omega_{\rm ci}}{\omega_1-n\omega_{\rm ci}}J_n^1f_{0i}^0, \quad (22)$$
$$f_{\rm ni}^2=\frac{e\phi_2}{T_i}\,\frac{n\omega_{\rm ci}}{\omega_2-n\omega_{\rm ci}}J_n^2f_{0i}^0.$$

Using Eq. (22) in Eq. (5), we obtain the nonlinear part of the distribution function for the low-frequency mode $(\omega,\mathbf{k})$ as

$$f_{\rm ni}^{\rm NL}=\frac{\exp(-in\delta)}{\omega-n\omega_{\rm ci}}(\psi_1+\psi_2), \quad (23)$$

where

$$\psi_1=\sum_l\bigg[\frac{ek_0\phi_0}{2m_iv_\perp}(n-l)\exp[i(n-l)\delta_1]J_l^{0\prime}f_{(n-l)i}^1+\frac{ek_{1\perp}\phi_1}{2m_iv_\perp}(n-l)\exp(il\delta_1)J_l^{1\prime}f_{(n-l)i}^0-\frac{e\phi_0l}{2}\exp[i(n-l)\delta_1]J_l^0\frac{\partial f_{(n-l)i}^1}{\partial\mu}$$
$$-\frac{e\phi_1l}{2}\exp(il\delta_1)J_l^1\frac{\partial f_{(n-l)i}^0}{\partial\mu}+\frac{iek_0k_{1\perp}\phi_0\sin\delta_1}{2m_i\omega_{\rm ci}}\exp[i(n-l)\delta_1]J_l^0f_{(n-1)i}^1-\frac{iek_0k_{1\perp}\phi_1\sin\delta_1}{2m_i\omega_{\rm ci}}\exp(il\delta_1)J_l^1f_{(n-l)i}^0\bigg], \quad (24)$$

$$\psi_2=\sum_l\bigg[\frac{ek_0\phi_0^*}{2m_iv_\perp}(n+l)\exp[i(n+l)\delta_2]J_l^{0\prime}f_{(n+l)i}^2-\frac{ek_{2\perp}\phi_2}{2m_iv_\perp}(l-n)\exp(il\delta_2)J_l^{2\prime}f_{(l-n)i}^{0*}+\frac{e\phi_0^*l}{2}\exp[i(n+l)\delta_2]J_l^0\frac{\partial f_{(n+l)i}^2}{\partial\mu}$$
$$-\frac{e\phi_2l}{2}\exp(il\delta_2)J_l^2\frac{\partial f_{(l-n)i}^{0*}}{\partial\mu}-\frac{iek_0k_{2\perp}\phi_0^*\sin\delta_2}{2m_i\omega_{\rm ci}}\exp[i(n+l)\delta_2]J_l^0f_{(n+l)i}^2+\frac{iek_0k_{2\perp}\phi_2\sin\delta_2}{2m_i\omega_{\rm ci}}\exp(il\delta_2)J_l^2f_{(l-n)i}^{0*}\bigg], \quad (25)$$

where the asterisk denotes the complex conjugate of the quantity involved. We obtain the linear and nonlinear ion density perturbations associated with low-frequency mode $(\omega,\mathbf{k})$ from the relation

$$n_i(\omega,\mathbf{k})=2\pi\sum_n\int_0^\infty\int_{-\infty}^\infty f_{ni}J_nv_\perp dv_\perp dv_\parallel. \quad (26)$$

Thus the linear and nonlinear ion density perturbations at $(\omega,\mathbf{k})$ are obtained as

$$n_i^L=-\frac{\chi_ik^2}{4\pi e}\phi \quad (27)$$

and

$$n_i^{\mathrm{NL}}=\frac{n_0^0 e^2 \exp(-i\delta)}{2m_i T_i(\omega-\omega_{\mathrm{ci}})}(\phi_0\phi_1 X_i+\phi_0^*\phi_2 Y_i), \tag{28}$$

where

$$\chi_i=\frac{2\omega_{\mathrm{pi}}^2}{k^2 v_{\mathrm{th},i}^2}\left[1+\frac{\omega}{k_\parallel v_{\mathrm{th},i}}\sum_n Z\left(\frac{\omega-n\omega_{\mathrm{ci}}}{k_\parallel v_{\mathrm{th},i}}\right)I_n(b_i)\exp(-b_i)\right], \tag{29}$$

$$X_i=\left[\frac{ik_0k_{1\perp}\sin\delta_1\exp(i\delta_1)}{\omega_1-\omega_{\mathrm{ci}}}I_0(b_{0i})\exp(-b_{0i})-\frac{ik_0k_{1\perp}\sin\delta_1}{\omega_0-\omega_{\mathrm{ci}}}I_0(b_{1i})\exp(-b_{1i})-\frac{m_i\omega_{\mathrm{ci}}^2\exp(i\delta_1)}{2T_i(\omega_1-\omega_{\mathrm{ci}})}\{1-I_0(b_{0i})\exp(-b_{0i})\}\right.$$
$$\left.-\frac{m_i\omega_{\mathrm{ci}}^2}{2T_i(\omega_0-\omega_{\mathrm{ci}})}\{1-I_0(b_{1i})\exp(-b_{1i})\}\right], \tag{30}$$

and

$$Y_i=\left[\frac{ik_0k_{2\perp}\sin\delta_2\exp(2i\delta_2)}{\omega_0-\omega_{\mathrm{ci}}}I_2(b_{2i})\exp(-b_{2i})-\frac{ik_0k_{2\perp}\sin\delta_2\exp(i\delta_2)}{\omega_2-\omega_{\mathrm{ci}}}I_0(b_{0i})\exp(-b_{0i})\right.$$
$$-\frac{m_i\omega_{\mathrm{ci}}^2\exp(i\delta_2)}{2T_i(\omega_2-\omega_{\mathrm{ci}})}\{1-I_0(b_{0i})\exp(-b_{0i})\}+\frac{m_i\omega_{\mathrm{ci}}^2\exp(2i\delta_2)}{2T_i(\omega_0-\omega_{\mathrm{ci}})}\{1-I_2(b_{2i})\exp(-b_{2i})\}$$
$$\left.+\frac{m_i\omega_{\mathrm{ci}}^2\exp(2i\delta_2)}{T_i(\omega_0-\omega_{\mathrm{ci}})}\{1+I_1(b_{0i})\exp(-b_{0i})\}\right]. \tag{31}$$

$v_{\mathrm{th},i}=(2T_i/m_i)^{1/2}$; I_0, I_1, and I_2 are the zero, first, and second order modified Bessel functions of the first kind; $b_{0i}=k_0^2 v_{\mathrm{th},i}^2/2\omega_{\mathrm{ci}}^2>1$ for the usual plasma parameters in the beat wave accelerators; $b_{1i}=k_1^2 v_{\mathrm{th},i}^2/2\omega_{\mathrm{ci}}^2>1$; and $b_{2i}=k_2^2 v_{\mathrm{th},i}^2/2\omega_{\mathrm{ci}}^2>1$, for the short-wavelength perturbation. In deriving Eq. (28) we have retained only the dominating terms having $(\omega-\omega_{\mathrm{ci}})$ in the denominator.

B. Response of electrons

For the low-frequency response of magnetized electrons we use the Vlasov equation in terms of the guiding center coordinates. Following the same procedure as in the case of the magnetized ions, we find the linear and nonlinear density perturbations of electrons as

$$n_e^L=\frac{\chi_e k^2}{4\pi e}\phi, \tag{32}$$

and

$$n_e^{\mathrm{NL}}=\frac{n_0^0 e^2\exp(i\delta)}{2m_e T_e(\omega-\omega_{\mathrm{ce}})}(\phi_0\phi_1 X_e+\phi_0^*\phi_2 Y_e), \tag{33}$$

where

$$\chi_e=\frac{2\omega_{\mathrm{pe}}^2}{k^2 v_{\mathrm{th},e}^2}\left[1+\frac{\omega}{k_\parallel v_{\mathrm{th},e}}\sum_n Z\left(\frac{\omega-n\omega_{\mathrm{ce}}}{k_\parallel v_{\mathrm{th},e}}\right)I_n(b_e)\exp(-b_e)\right], \tag{34}$$

$$X_e=\left[-\frac{m_e\omega_{\mathrm{ce}}^2\exp(-i\delta_1)}{2T_e(\omega_1-\omega_{\mathrm{ce}})}\{1-I_0(b_{0e})\exp(-b_{0e})\}-\frac{m_e\omega_{\mathrm{ce}}^2}{2T_e(\omega_0-\omega_{\mathrm{ce}})}\{1-I_0(b_{1e})\exp(-b_{1e})\}\right.$$
$$\left.+\frac{ik_0k_{1\perp}\sin\delta_1}{\omega_0-\omega_{\mathrm{ce}}}I_0(b_{1e})\exp(-b_{1e})-\frac{ik_0k_{1\perp}\sin\delta_1\exp(-i\delta_1)}{\omega_1-\omega_{\mathrm{ce}}}I_0(b_{0e})\exp(-b_{0e})\right], \tag{35}$$

$$Y_e=\left[\frac{m_e\omega_{\rm ce}^2\exp(-2i\delta_2)}{2T_e(\omega_0-\omega_{\rm ce})}\{1-I_2(b_{2e})\exp(-b_{2e})\}-\frac{m_e\omega_{\rm ce}^2\exp(-i\delta_2)}{2T_e(\omega_2-\omega_{\rm ce})}\{1-I_0(b_{0e})\exp(-b_{0e})\}\right.$$
$$+\frac{m_e\omega_{\rm ce}^2\exp(-2i\delta_2)}{T_e(\omega_0-\omega_{\rm ce})}\{1+I_1(b_{0e})\exp(-b_{0e})\}+\frac{ik_0k_{2\perp}\sin\delta_2\exp(-2i\delta_2)}{\omega_0-\omega_{\rm ce}}I_1(b_{0e})\exp(-b_{0e})$$
$$\left.+\frac{ik_0k_{2\perp}\exp(-i\delta_2)}{\omega_2-\omega_{\rm ce}}I_0(b_{0e})\exp(-b_{0e})\right]. \qquad (36)$$

In the case of the plasma beat wave accelerators, the condition $\omega_{\rm pe}^2>\omega_{\rm ce}^2$ is always satisfied and we may consider the high-frequency response of electrons to be unmagnetized. Taking the high-frequency response of electrons to be unmagnetized, the solution of the linearized Vlasov equation for the response at $(\omega_0,\mathbf{k}_0)$ may be written as

$$f_0^L=-\frac{e\phi_0}{T_e}\frac{\mathbf{k}_0\cdot\mathbf{v}}{\omega_0}\left(1+\frac{\mathbf{k}_0\cdot\mathbf{v}}{\omega_0}\right)f_0^0, \qquad (37)$$

where $\omega_0>\mathbf{k}_0\cdot\mathbf{v}$ is assumed. For the high-frequency response at $(\omega_1,\mathbf{k}_1)$ and $(\omega_2,\mathbf{k}_2)$ we express f_1 and f_2 as

$$f_1=f_1^L+f_1^{\rm NL},$$
$$f_2=f_2^L+f_2^{\rm NL}, \qquad (38)$$

where the linear and nonlinear parts of the distribution functions f_1^L, f_2^L, $f_1^{\rm NL}$, and $f_2^{\rm NL}$ in the limit $\omega_{1,2}>\mathbf{k}_{1,2}\cdot\mathbf{v}$ turn out to be

$$f_{1,2}=-\frac{e\phi_{1,2}}{T_e}\frac{\mathbf{k}_{1,2}\cdot\mathbf{v}}{\omega_{1,2}}\left(1+\frac{\mathbf{k}_{1,2}\cdot\mathbf{v}}{\omega_{1,2}}\right)f_0^0, \qquad (39)$$

$$f_1^{\rm NL}=-\frac{e}{2m_e\omega_1}\left(1+\frac{\mathbf{k}_1\cdot\mathbf{v}}{\omega_1}\right)(\mathbf{k}_0\cdot\nabla_v f^L\phi_0^*-\mathbf{k}\cdot\nabla_v f_0^{L*}\phi), \qquad (40)$$

$$f_2^{\rm NL}=\frac{e}{2m_e\omega_2}\left(1+\frac{\mathbf{k}_2\cdot\mathbf{v}}{\omega_2}\right)(\mathbf{k}_0\cdot\nabla_v f^L\phi_0+\mathbf{k}\cdot\nabla_v f_0^L\phi). \qquad (41)$$

Integrating $f_1^{\rm NL}$ and $f_2^{\rm NL}$ in velocity space, we obtain the following expressions for the nonlinear density fluctuations at the high-frequency sidebands $(\omega_1,\mathbf{k}_1)$ and $(\omega_2,\mathbf{k}_2)$ for motion of electrons:

$$n_{1e}^{\rm NL}=\frac{e}{2m_e\omega_1^2}[(\mathbf{k}_0\cdot\mathbf{k}_1)\phi_0^*n_e^L-(\mathbf{k}\cdot\mathbf{k}_1)\phi n_0^{L*}], \qquad (42)$$

$$n_{2e}^{\rm NL}=\frac{e}{2m_e\omega_2^2}[(\mathbf{k}_0\cdot\mathbf{k}_2)\phi_0n_e^L+(\mathbf{k}\cdot\mathbf{k}_2)\phi n_0^{L*}], \qquad (43)$$

where n_e^L is the linear density perturbation at the low frequency given by Eq. (32), and n_0^L is the linear density fluctuation associated with the pump wave:

$$n_0^L=-\frac{ek_0^2\phi_0n_0^0}{m_e\omega_0^2}. \qquad (44)$$

Now, substituting Eqs. (27), (28), (32), (33), (42), (43), and (44) in the Poisson's equation, we obtain the following nonlinear coupled equations:

$$\epsilon\phi=-\frac{4\pi e}{k^2}(n_e^{\rm NL}-n_i^{\rm NL}), \qquad (45)$$

$$\epsilon_1\phi_1=-\frac{4\pi e}{k_1^2}n_{1e}^{\rm NL}, \qquad (46)$$

$$\epsilon_2\phi_2=-\frac{4\pi e}{k_2^2}n_{2e}^{\rm NL}, \qquad (47)$$

where the linear dielectric functions ϵ, ϵ_1, and ϵ_2 are given by

$$\epsilon=1+\chi_e+\chi_i$$
$$=1+\frac{2\omega_{\rm pe}^2}{k^2v_{{\rm th},e}^2}\left[1+\frac{\omega}{k_\parallel v_{{\rm th},e}}\sum_n Z\left(\frac{\omega-n\omega_{\rm ce}}{k_\parallel v_{{\rm th},e}}\right)I_n(b_e)\exp(-b_e)\right]$$
$$+\frac{2\omega_{\rm pi}^2}{k^2v_{{\rm th},i}^2}\left[1+\frac{\omega}{k_\parallel v_{{\rm th},i}}\sum_n Z\left(\frac{\omega-n\omega_{\rm ci}}{k_\parallel v_{{\rm th},i}}\right)I_n(b_i)\exp(-b_i)\right], \qquad (48)$$

$$\epsilon_1=1+\frac{2\omega_{\rm pe}^2}{k_1^2v_{{\rm th},e}^2}\left[1+\frac{\omega_1}{k_{1\parallel}v_{{\rm th},e}}\sum_n Z\left(\frac{\omega_1-n\omega_{\rm ce}}{k_{1\parallel}v_{{\rm th},e}}\right)\times I_n(b_{1e})\exp(-b_{1e})\right], \qquad (49)$$

$$\epsilon_2=1+\frac{2\omega_{\rm pe}^2}{k_2^2v_{{\rm th},e}^2}\left[1+\frac{\omega_2}{k_{2\parallel}v_{{\rm th},e}}\sum_n Z\left(\frac{\omega_2-n\omega_{\rm ce}}{k_{2\parallel}v_{{\rm th},e}}\right)\times I_n(b_{2e})\exp(-b_{2e})\right]. \qquad (50)$$

We have taken the response of electrons and ions for the low-frequency mode $(\omega,\mathbf{k})$, while only the electron response has been taken into account for the high-frequency modes, where the ions form only the charge neutralizing background. Eliminating ϕ, ϕ_1, and ϕ_2 from Eqs. (45)–(47) we obtain the following expression for the nonlinear dispersion relation for the low-frequency electrostatic mode $(\omega,\mathbf{k})$:

$$\epsilon=\frac{\mu_1}{\epsilon_1}+\frac{\mu_2}{\epsilon_2}, \qquad (51)$$

where

$$\mu_1=\frac{\omega_{\rm pe}^2 e^2\phi_0^*\phi_0}{4m_eT_ek^2k_1^2\omega_1^2(\omega-\omega_{\rm ce})}\left(\chi_e k^2\mathbf{k}_0\cdot\mathbf{k}_1+k_0^2\frac{\omega_{\rm pe}^2}{\omega_0^2}\mathbf{k}\cdot\mathbf{k}_1\right)\times\left[X_e\,\exp(i\delta)-\frac{m_eT_e(\omega-\omega_{\rm ce})}{m_iT_i(\omega-\omega_{\rm ci})}X_i\,\exp(-i\delta)\right] \quad (52)$$

and

$$\mu_2=\frac{\omega_{\rm pe}^2 e^2\phi_0^*\phi_0}{4m_eT_ek^2k_2^2\omega_2^2(\omega-\omega_{\rm ce})}\left(\chi_e k^2\mathbf{k}_0\cdot\mathbf{k}_2-k_0^2\frac{\omega_{\rm pe}^2}{\omega_0^2}\mathbf{k}\cdot\mathbf{k}_2\right)\times\left[Y_e\,\exp(i\delta)-\frac{m_eT_e(\omega-\omega_{\rm ce})}{m_iT_i(\omega-\omega_{\rm ci})}Y_i\,\exp(-i\delta)\right]. \quad (53)$$

This is the general dispersion relation of any low-frequency electrostatic mode $(\omega,\mathbf{k})$ in the presence of an electrostatic pump wave and the sidebands in a transversely magnetized plasma. From Eqs. (52) and (53), we notice that the effect of ion motion enters into the coupling coefficients μ_1 and μ_2 through the terms containing X_i and Y_i. For $\omega\sim\omega_{\rm ci}$, the ion contribution may dominate over that due to motion of electrons.

III. EXPRESSION FOR GROWTH RATE

To obtain the growth rate of the decay process we write [14,16]

$$\omega=\omega_r+i\gamma,$$

$$\epsilon=i\gamma\frac{\partial\epsilon_r}{\partial\omega}+i\epsilon_i=i(\gamma+\gamma_L)\frac{\partial\epsilon_r}{\partial\omega}, \quad (54)$$

where the subscript r represents a real quantity and γ_L, the linear damping rate of the low-frequency mode may be obtained from the relation

$$\gamma_L=\frac{-\epsilon_i}{\partial\epsilon_r/\partial\omega}. \quad (55)$$

Similarly

$$\epsilon_1=i(\gamma+\gamma_{L1})\frac{\partial\epsilon_{1r}}{\partial\omega_1}, \quad (56)$$

$$\epsilon_2=i(\gamma+\gamma_{L2})\frac{\partial\epsilon_{2r}}{\partial\omega_2}, \quad (57)$$

where γ_{L1} and γ_{L2} are the linear damping rates of the decay waves $(\omega_1,\mathbf{k}_1)$ and $(\omega_2,\mathbf{k}_2)$.

Thus the growth rate of the four-wave parametric instability is obtained from

$$(\gamma+\gamma_L)(\gamma+\gamma_{L1})(\gamma+\gamma_{L2})\equiv-\frac{1}{\partial\epsilon/\partial\omega}\left[\frac{\mu_1(\gamma+\gamma_{L2})}{\partial\epsilon_1/\partial\omega_1}+\frac{\mu_2(\gamma+\gamma_{L1})}{\partial\epsilon_2/\partial\omega_2}\right], \quad (58)$$

where γ is the growth rate in presence of the damping of the waves. Now the linear dielectric function of the low-frequency mode $(\omega,\mathbf{k})$ propagating along the x direction can be simplified with the approximations $\omega\sim\omega_{\rm ci}$ and $\omega_0,\omega_1,\omega_2\gg\mathbf{k}_{0,1,2}\cdot\mathbf{v}$, and $\delta=\delta_1=\delta_2=0$ (when all waves are considered to be propagating along the x direction), Eq. (48) becomes

$$\epsilon=1+\frac{2\omega_{\rm pe}^2}{k^2v_{{\rm th},e}^2}+\frac{2\omega_{\rm pi}^2}{k^2v_{{\rm th},i}^2}\left[1-\frac{\omega}{\omega-\omega_{\rm ci}}\frac{1}{\sqrt{(2\pi b_i)}}\right] \quad (59)$$

and

$$\frac{\partial\epsilon}{\partial\omega}=\frac{2}{\sqrt{\pi}}\frac{\omega_{\rm pi}^2\omega_{\rm ci}^2}{k^3v_{{\rm th},i}^3(\omega-\omega_{\rm ci})^2}. \quad (60)$$

Using Eqs. (49), (50), (52), (53), and (60) we obtain the normalized growth rate $(\gamma\equiv\gamma_0)$ of the modulational instability in the absence of the linear damping of all the decay waves $(\gamma_L=\gamma_{L1}=\gamma_{L2}=0)$ as

$$\frac{\gamma_0}{\omega}=\left[\frac{|v_0/v_{{\rm th},e}|^2\pi k^3v_{{\rm th},e}^3v_{{\rm th},i}^3(\omega-\omega_{\rm ci})^2\omega_0^2}{8k_0\omega_{\rm ce}^2\omega_{\rm pi}^2\omega_{\rm ci}^2(\omega-\omega_{\rm ce})\omega^2}\left\{\frac{k_1^2(\omega_1-\omega_{\rm ce})^2}{\omega_1^2}\left(\chi_ek+k_0\frac{\omega_{\rm pe}^2}{\omega_0^2}\right)\left(A-\frac{m_eT_e(\omega-\omega_{\rm ce})}{m_iT_i(\omega-\omega_{\rm ci})}B\right)+\frac{k_2^2(\omega_2-\omega_{\rm ce})^2}{\omega_2^2}\left(\chi_ek-k_0\frac{\omega_{\rm pe}^2}{\omega_0^2}\right)\left(C-\frac{m_eT_e(\omega-\omega_{\rm ce})}{m_iT_i(\omega-\omega_{\rm ci})}D\right)\right\}\right]^{1/2}, \quad (61)$$

where

$$v_0^2=\frac{e^2k_0^2\phi_0^*\phi_0}{m_e^2\omega_0^2}, \quad (62)$$

$$A=\frac{m_e\omega_{\rm ce}^2}{2T_e(\omega_1-\omega_{\rm ce})}[1-I_0(b_{0e})\exp(-b_{0e})]+\frac{m_e\omega_{\rm ce}^2}{2T_e(\omega_0-\omega_{\rm ce})}[1-I_0(b_{1e})\exp(-b_{1e})], \quad (63)$$

$$B=\frac{m_i\omega_{\rm ci}^2}{2T_i(\omega_1-\omega_{\rm ci})}[1-I_0(b_{0i})\exp(-b_{0i})]+\frac{m_i\omega_{\rm ci}^2}{2T_i(\omega_0-\omega_{\rm ci})}[1-I_0(b_{1i})\exp(-b_{1i})], \quad (64)$$

$$C=-\frac{m_e\omega_{ce}^2}{2T_e(\omega_0-\omega_{ce})}[1-I_2(b_{2e})\exp(-b_{2e})]-\frac{m_e\omega_{ce}^2}{T_e(\omega_0-\omega_{ce})}[1-I_1(b_{0e})\exp(-b_{0e})]+\frac{m_e\omega_{ce}^2}{T_e(\omega_2-\omega_{ce})}[1-I_0(b_{0e})\exp(-b_{0e})], \tag{65}$$

$$D=-\frac{m_i\omega_{ci}^2}{2T_i(\omega_0-\omega_{ci})}[1-I_2(b_{2i})\exp(-b_{2i})]+\frac{m_i\omega_{ci}^2}{2T_i(\omega_2-\omega_{ce})}[1-I_0(b_{0i})\exp(-b_{0i})]-\frac{m_i\omega_{ci}^2}{T_i(\omega_0-\omega_{ci})}[1-I_1(b_{0i})\exp(-b_{0i})]. \tag{66}$$

Since we are considering all the waves propagating in the x direction, $\delta=\delta_1=\delta_2=0$ and $k_z=k_{1z}=k_{2z}=0$. Hence there is no linear damping of the waves. Consequently, the threshold of the modulational instability of the electrostatic beat wave is zero. Thus the undamped growth rate given by Eq. (61) is due only to the nonlinear beating of the pump and the sidebands. It may be anticipated that the motion of ions to the undamped growth rate of the modulational instability [Eq. (61)] may contribute significantly and even dominate over the contribution of electron motion for $\omega\sim\omega_{ci}$.

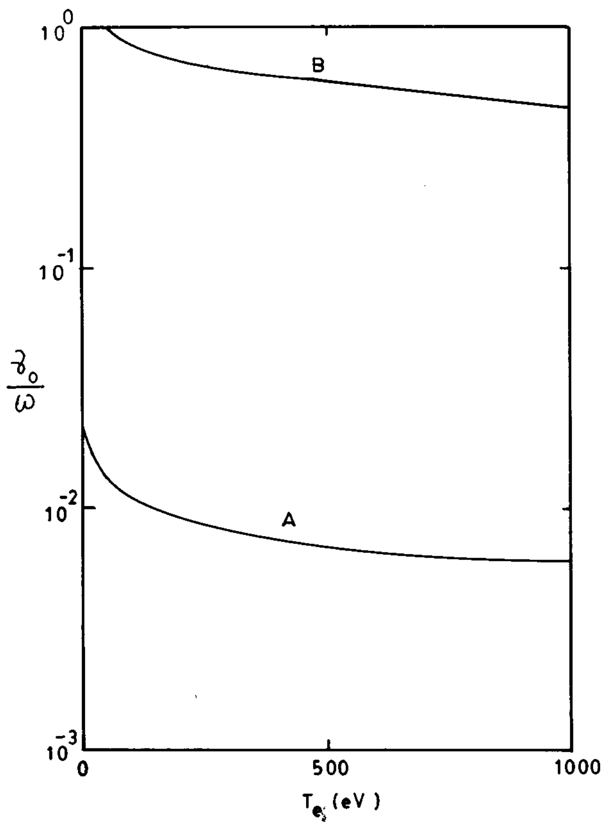

FIG. 1. Variation of γ_0/ω with T_e for the following parameters: $|v_0/v_{th,e}|=1.0$, $T_e=1$ keV, $T_e/T_i=100$, $B_s=20$ kG, $\omega_0=1.85\times10^{13}$ rad sec^{-1}, $\omega/\omega_{ci}=1.001$, and $n_0^0=10^{17}$ cm^{-3}. Curve A corresponds to the normalized growth rate for the electron motion only. Curve B corresponds to the normalized growth rate of the instability when both electron and ion motions are included.

IV. NUMERICAL RESULTS AND GRAPHICAL REPRESENTATIONS

To have some numerical appreciation of the results of our theory, we have made calculations for the growth rate of modulational instability for the following typical plasma parameters: $\omega_1'=1.963\times10^{14}$ rad sec^{-1}, $\omega_2'=1.778\times10^{14}$ rad sec^{-1} (corresponding to a CO_2 laser), $n_0^0=10^{17}$ cm^{-3}, $T_e=1$ keV, $B_s=20$–200 kG, $|v_0/v_{th,e}|=1.0$, $\omega/\omega_{ci}=1.001$, and $T_e/T_i=100$. We have chosen the above set of parameters for our calculations, because of their relevance to plasma beat wave experimental studies. The results of calculations are presented in the form of graphs in Figs. 1–4.

Figure 1 shows the variation of the normalized growth rate γ_0/ω for the modulational instability with electron plasma temperature T_e. From Fig. 1, we note that modula-

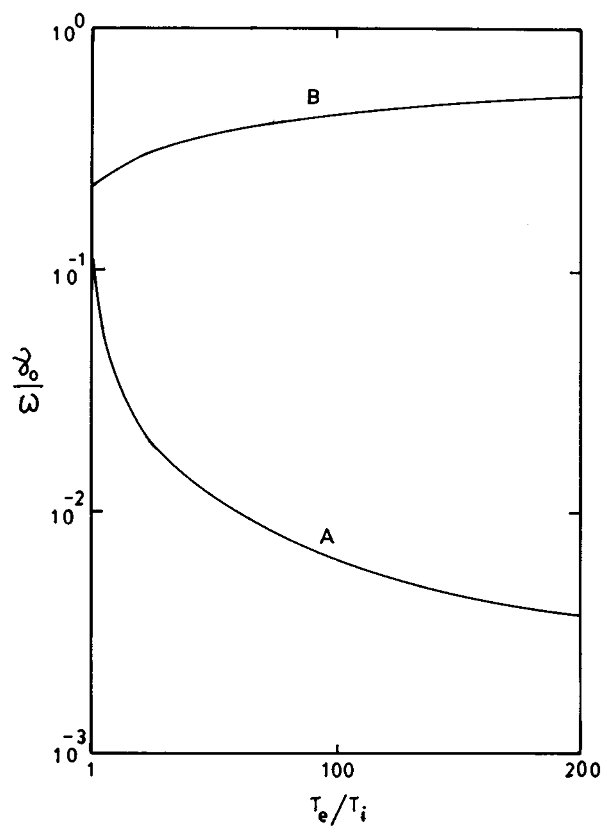

FIG. 2. Variation of γ_0/ω with T_e/T_i for the following parameters: $|v_0/v_{th,e}|=1.0$, $T_e=1$ keV, and $B_s=20$ kG. The other parameters and specifications are the same as in Fig. 1.

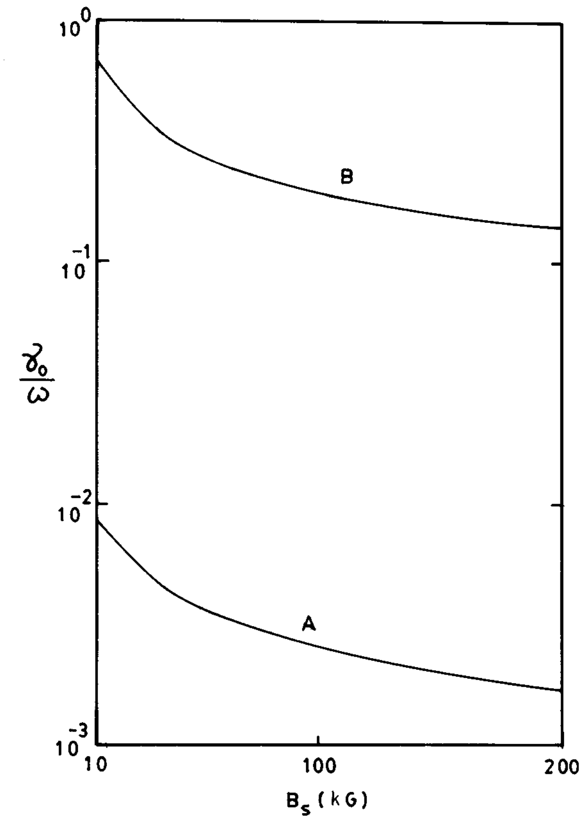

FIG. 3. Variation of γ_0/ω with the external magnetic field B_s for the following parameters: $|v_0/v_{\mathrm{th},e}|=1.0$, $T_e/T_i=100$, and $T_e=1$ keV. The other parameters and specifications are the same as in Fig. 1.

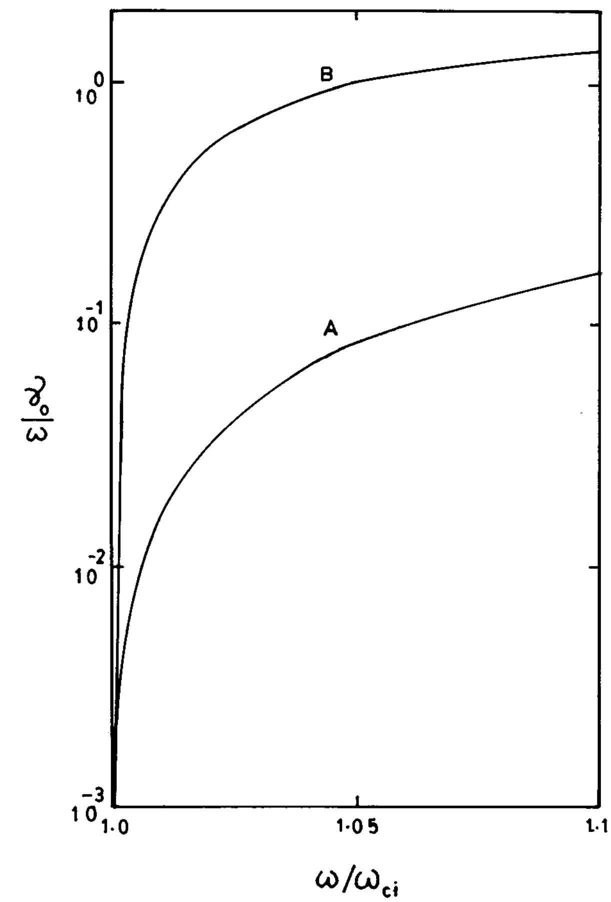

FIG. 4. Variation of γ_0/ω with $\omega/\omega_{\mathrm{ci}}$ for the following parameters: $|v_0/v_{\mathrm{th},e}|=1.0$, $T_e/T_i=100$, $T_e=1$ keV, and $B_s=200$ kG. The other parameters and specifications are the same as in Fig. 1.

tional instability decreases at low temperature but becomes more or less independent of T_e for a given T_e/T_i.

Figure 2 shows the variation of the normalized growth rate γ_0/ω for the modulational instability of the beat wave with T_e/T_i. It is noticed that the modulational instability decreases rapidly with T_e/T_i only when electron motion is considered; but with ion motion included, the modulational instability takes a steady value.

Figure 3 shows the variation of the normalized growth rate γ_0/ω of the modulational instability of the beat wave with external magnetic field B_s. It follows that the normalized growth rate of the modulational instability decreases slowly with increasing B_s. However, the growth rate is about two orders higher when ion motion is included.

Figure 4 shows the variation of the growth rate γ_0/ω with $\omega/\omega_{\mathrm{ci}}$. We notice that the growth rate increases with $\omega/\omega_{\mathrm{ci}}$. When ion motion is included, the growth rate of the instability takes a higher steady value. For unmagnetized plasmas [16], the growth rate of the modulational instability is quite high (in the order of $\sim\omega_{\mathrm{pi}}$). But, due to the application of the external static magnetic field in the present investigation, the growth rate of the modulational instability reduces substantially, on the order of $\sim\omega_{\mathrm{ci}}\ll\omega_{\mathrm{pi}}$.

V. DISCUSSION

In this paper, we have studied the modulational instability of an electrostatic electron plasma wave excited by two electromagnetic waves in a transversely magnetized plasma. The transverse static external magnetic field is applied with a view to phase lock the accelerated particles with the wave; that is, preventing these particles from outrunning the plasma beat wave, thereby allowing the particles to gain maximum energy. The nonlinear response of electrons and ions has been obtained by solving the full Vlasov equation expressed in terms of gyrokinetic variables. It is seen from the expression for the growth rate of the modulational instability [cf. Eq. (61)] that the growth rate vanishes when $\omega=\omega_{\mathrm{ci}}$ because at the ion cyclotron resonance of the low-frequency mode $(\omega,\mathbf{k})$ the wave is totally damped to the ions. For unmagnetized plasmas [16], the growth rate of the modulational instability is quite high (on the order of $\sim\omega_{\mathrm{pi}}$). But, due to the application of the external static magnetic field in the present investigation, the growth rate of the modulational instability reduces substantially, on the order of $\sim\omega_{\mathrm{ci}}\ll\omega_{\mathrm{pi}}$. The ion nonlinearity is seen to dominate over the electron nonlinearity for the low-frequency perturbation for $\omega\sim\omega_{\mathrm{ci}}$. From the numerical calculations, we observe that the growth rate of the modulational instability is about two orders higher when ion motion is included. We note that the modulational instability decreases at low temperature but becomes more or less independent of T_e for a given T_e/T_i. It is also noted that the modulational instability decreases rapidly with T_e/T_i only when electron motion is considered; however, with ion motion included, the modulational instability takes a steady value. We also note that the growth rate of the modulational instability decreases slowly with increasing B_s. Therefore, the application of an external magnetic field in the plasma beat wave acceleration scheme is also important for suppressing modulational instability which otherwise may destroy the acceleration mechanism [9,14].

It may be mentioned further that the effect of ion dynamics on other parametric instabilities, such as decay, oscillating two-steam instabilities, etc. of the longitudinal beat wave may become important and significant. Work along this line is in progress.

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